

MULTIMEDIA



UNIVERSITY

STUDENT ID NO

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# MULTIMEDIA UNIVERSITY

## FINAL EXAMINATION

TRIMESTER 2, 2015/2016

**BFS1024 – STATISTICS FOR FINANCE**

( All sections / Groups )

8 MARCH 2016  
2.30 p.m. - 4.30 p.m.  
( 2 Hours )

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### INSTRUCTIONS TO STUDENTS

1. This question paper consists of **THREE (3)** printed pages exclusive of the cover page, formulae sheet and statistical tables.
2. Answer **ALL** four questions in the answer booklet provided.
3. Students are allowed to use non-programmable scientific calculators only.
4. Marks are shown at the end of each question.

**Question 1 [27 marks]**

- (a) An industrial plant is conducting a study to determine how quickly injured workers are back on the job following injury. Records show that 10% of all injured workers are admitted to the hospital for treatment and 15% are back on the job the next day. In addition, studies show that 2% are admitted for hospital treatment and back on the job the next day.
- If a worker is injured, what is the probability that the worker will either be admitted to a hospital or be back on the job the next day or both? [3 marks]
  - What is the probability that an admitted worker not to be back on job the next day? [6 marks]
- (b) Amber operates a pastry shop selling two of their signature cake, the strawberry and chocolate cakes. Let  $X$  denotes the number of strawberry cakes sold daily and  $Y$  denotes the number of chocolate cakes sold daily. The distribution of the data is provided as follows.

$x$	15	18	20
$P(x)$	0.50	0.30	0.20

$y$	18	22	32
$P(y)$	0.60	0.30	0.10

- Find the expected number and variance of strawberry cakes sold daily. [6 marks]
- Find the expected number and variance of chocolate cakes sold daily. [6 marks]
- Find the mean and variance of total cakes sold daily assuming sales of both cakes are not correlated. [6 marks]

**Question 2 [25 marks]**

- (a) A local drugstore owner knows that, on average, 50 people enter his store each hour.
- Find the probability that in a given 3-minute period, two people enter the store. [4 marks]
  - Find the probability that at most 2 people enters the store for the first 5 minutes after opening store. [8 marks]

Continued...

- (b) The finished inside diameter of a piston ring is normally distributed with a mean of 10 centimeters and a standard deviation of 0.3 millimeter.
- What proportion of rings will have inside diameter exceeding 10.07 centimeters? [3 marks]
  - What is the probability that a piston ring will have an inside diameter between 9.9 and 9.95 centimeters? [6 marks]
  - Below what value of inside diameter will 15% of the piston rings fall? [4 marks]

**Question 3 [19 marks]**

A study of the effects of smoking on sleep patterns is conducted. The measure observed is the time, in minutes, that it takes to fall asleep. The following data are obtained as follows.

Smokers	Non-smokers
69.3	28.6
56.0	29.8
22.1	30.6
47.6	36.0
53.2	25.1
48.1	28.4
52.7	31.8
34.4	37.9
60.2	
43.8	

- Determine the point estimate of population mean and standard deviation for both the smokers and non-smokers samples. [6 marks]
- Can we conclude that the smokers having more difficulties in falling asleep at 5% significant level? [13 marks]

Continued...

**Question 4 [29 marks]**

A study was done on a diesel-powered light-duty pickup truck to see if humidity, air temperature and barometric pressure influence emission of nitrous oxide (in ppm). Emission measurements were taken at different times, with varying experimental conditions and partial output was provided.

ANOVA	df	SS	MS
Regression		3605.7736	
Error			
<b>Total</b>	<b>49</b>	<b>4820</b>	

Variable	Coefficients	p-value
Intercept	– 3.508	
Humidity	– 0.003	0.005
Temperature	0.001	0.011
Pressure	0.154	0.053

- (a) Determine the regression equation. [3 marks]
- (b) Interpret the impact of humidity on the emission of nitrous oxide. [3 marks]
- (c) Test whether the model is significant. [15 marks]
- (d) Determine which of the predictors to have significant impact on the emission of nitrous oxide at 5% significant level. [8 marks]

**End of Page**

**A. DESCRIPTIVE STATISTICS**

$$\text{Mean} = \frac{\sum X_i}{n}$$

$$\text{Standard Deviation (s)} = \sqrt{\frac{\sum X^2}{n-1} - \frac{(\sum X)^2}{n(n-1)}}$$

$$\text{Pearson's Coefficient of Skewness (S}_k\text{)} = \frac{3(\bar{X} - \text{Median})}{s} \text{ or } \frac{\bar{X} - \text{Mode}}{s}$$

**B. PROBABILITY**

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A \text{ and } B) = P(A) P(B) \text{ if } A \text{ and } B \text{ are independent}$$

$$P(A | B) = P(A \text{ and } B) / P(B)$$

**Poisson Probability Distribution**

$$\text{If } X \text{ follows a Poisson Distribution } P(\lambda) \text{ where } P(X = x) = \frac{e^{-\lambda} \lambda^x}{x!}$$

$$\text{then the mean} = E(X) = \lambda \text{ and variance} = \text{VAR}(X) = \lambda$$

**Binomial Probability Distribution**

$$\text{If } X \text{ follows a Binomial Distribution } B(n, p) \text{ where } P(X = x) = {}^n C_x p^x q^{n-x}$$

$$\text{then the mean} = E(X) = np \text{ and variance} = \text{VAR}(X) = npq \text{ where } q = 1 - p$$

**Normal Distribution**

$$\text{If } X \text{ follows a Normal distribution } N(\mu, \sigma) \text{ where } E(X) = \mu \text{ and } \text{VAR}(X) = \sigma^2$$

$$\text{then } z = \frac{X - \mu}{\sigma}$$

**C. EXPECTATION AND VARIANCE OPERATORS**

$$E(X) = \sum [X \cdot P(X)]$$

$$\text{VAR}(X) = E(X^2) - [E(X)]^2$$

$$\text{If } E(X) = \mu \text{ then } E(kX) = k\mu, E(X + Y) = E(X) + E(Y)$$

$$\text{If } \text{VAR}(X) = \sigma^2 \text{ then } \text{VAR}(kX) = k^2 \sigma^2,$$

$$\text{VAR}(aX + bY) = a^2 \text{VAR}(X) + b^2 \text{VAR}(Y) + 2ab \text{COV}(X, Y)$$

$$\text{where } \text{COV}(X, Y) = E(XY) - [E(X)E(Y)]$$

#### D. CONFIDENCE INTERVAL ESTIMATION AND SAMPLE SIZE DETERMINATION

$(100 - \alpha) \% \text{ Confidence Interval for Population Mean } (\sigma \text{ Known}) = \bar{X} \pm Z_{\alpha/2} \left( \frac{\sigma}{\sqrt{n}} \right)$
$(100 - \alpha) \% \text{ Confidence Interval for Population Mean } (\sigma \text{ Unknown}) = \bar{X} \pm t_{\alpha/2, n-1} \left( \frac{s}{\sqrt{n}} \right)$
$(100 - \alpha) \% \text{ Confidence Interval for Population Proportion} = p \pm Z_{\alpha/2} \sqrt{\frac{p(1-p)}{n}}$
$\text{Sample Size Determination for Population Mean} = n \geq \frac{(Z_{\alpha/2})^2 \sigma^2}{E^2}$
$\text{Sample Size Determination for Population Proportion} = n \geq \frac{(Z_{\alpha/2})^2 p(1-p)}{E^2}$
Where E = Limit of Error in Estimation

#### E. HYPOTHESIS TESTING

One Sample Mean Test	
Standard Deviation ( $\sigma$ ) Known	Standard Deviation ( $\sigma$ ) Not Known
$Z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$	$t = \frac{\bar{x} - \mu}{s / \sqrt{n}}$
One Sample Proportion Test	
$Z = \frac{p - \pi}{\sqrt{\frac{\pi(1-\pi)}{n}}}$	

**Two Sample Mean Test****Standard Deviation ( $\sigma$ ) Known**

$$z = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}}$$

**Standard Deviation ( $\sigma$ ) Not Known**

$$t = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{S_p^2 \left( \frac{1}{n_1} + \frac{1}{n_2} \right)}} \quad \text{where} \quad S_p^2 = \frac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{(n_1 - 1) + (n_2 - 1)}$$

**Two Sample Proportion Test**

$$z = \frac{(p_1 - p_2)}{\sqrt{p(1-p) \left[ \frac{1}{n_1} + \frac{1}{n_2} \right]}} \quad \text{where} \quad p = \frac{(n_1 p_1) + (n_2 p_2)}{n_1 + n_2} = \frac{X_1 + X_2}{n_1 + n_2}$$

where  $X_1$  and  $X_2$  are the number of successes from each population

**F. REGRESSION ANALYSIS****SIMPLE LINEAR REGRESSION:****Correlation Coefficient**

$$r = \frac{\sum XY - \left[ \frac{\sum X \sum Y}{n} \right]}{\sqrt{\left[ \sum X^2 - \left( \frac{(\sum X)^2}{n} \right) \right] \left[ \sum Y^2 - \left( \frac{(\sum Y)^2}{n} \right) \right]}} = \frac{COV(X, Y)}{\sigma_X \sigma_Y}$$

**Regression Coefficient**

$$b_1 = \frac{\sum XY - \left[ \frac{\sum X \sum Y}{n} \right]}{\left[ \sum X^2 - \left( \frac{(\sum X)^2}{n} \right) \right]}, \quad b_0 = \bar{Y} - b_1 \bar{X}$$

**MULTIPLE LINEAR REGRESSION:**

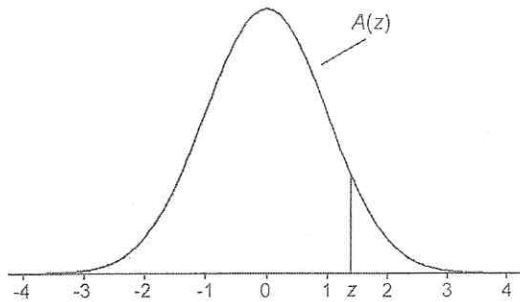
Adjusted r-square = $1 - \left[ \frac{(1 - r^2)(n - 1)}{(n - p - 1)} \right]$ where $p$ = number of independent variables
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ANOVA Table for Multiple Linear Regression			
Source	Degrees of Freedom	Sum of Squares	Mean Squares
Regression	$p$	SSR	$MSR = SSR/p$
Error	$n - p - 1$	SSE	$MSE = SSE/(n - p - 1)$
Total	$n - 1$	SST	
Test Statistic for Significance of the Overall Regression Model = $F = MSR/MSE$			
Test Statistic for Significance of each Explanatory Variable = $t^* = b_i / S_{bi}$ and the Critical $t = t_{(n - p - 1), \alpha/2}$			



TABLE A.1

## Cumulative Standardized Normal Distribution



$A(z)$  is the integral of the standardized normal distribution from  $-\infty$  to  $z$  (in other words, the area under the curve to the left of  $z$ ). It gives the probability of a normal random variable not being more than  $z$  standard deviations above its mean. Values of  $z$  of particular importance:

$z$	$A(z)$	
1.645	0.9500	Lower limit of right 5% tail
1.960	0.9750	Lower limit of right 2.5% tail
2.326	0.9900	Lower limit of right 1% tail
2.576	0.9950	Lower limit of right 0.5% tail
3.090	0.9990	Lower limit of right 0.1% tail
3.291	0.9995	Lower limit of right 0.05% tail

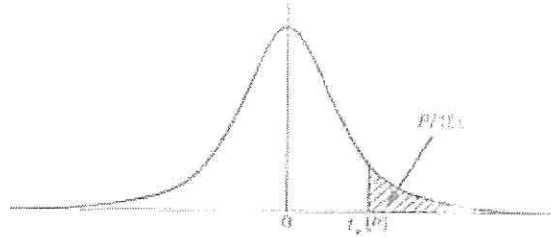
$z$	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999							

TABLE 10. PERCENTAGE POINTS OF THE *t*-DISTRIBUTION

This table gives percentage points  $t_p(P)$  defined by the equation

$$\frac{P}{100} = \frac{1}{\sqrt{\pi n}} \frac{\Gamma(\frac{1}{2}n + \frac{1}{2})}{\Gamma(\frac{1}{2}n)} \int_{t_p(P)}^{\infty} \frac{dt}{(1+t^2/\nu)^{\frac{1}{2}(\nu+1)}}.$$

Let  $X_1$  and  $X_2$  be independent random variables having a normal distribution with zero mean and unit variance and a  $\chi^2$ -distribution with  $\nu$  degrees of freedom respectively; then  $t = X_1/\sqrt{X_2/\nu}$  has Student's *t*-distribution with  $\nu$  degrees of freedom, and the probability that  $t \geq t_p(P)$  is  $P/100$ . The lower percentage points are given by symmetry as  $-t_p(P)$ , and the probability that  $|t| \geq t_p(P)$  is  $2P/100$ .

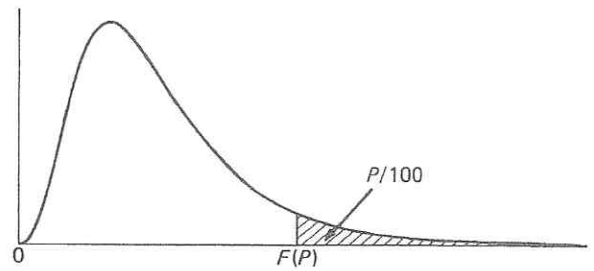


The limiting distribution of *t* as  $\nu$  tends to infinity is the normal distribution with zero mean and unit variance. When  $\nu$  is large interpolation in  $\nu$  should be harmonic.

P	40	30	25	20	15	10	5	2.5	1	0.5	0.1	0.05
$\nu = 1$	0.3249	0.7265	1.0000	1.3764	1.963	3.078	6.314	12.71	31.82	63.66	318.3	636.6
2	0.2887	0.6172	0.8165	1.0607	1.386	1.886	2.920	4.303	6.965	9.925	22.33	31.60
3	0.2767	0.5844	0.7649	0.9785	1.250	1.638	2.353	3.182	4.541	5.841	10.21	12.92
4	0.2707	0.5686	0.7407	0.9410	1.190	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.2672	0.5594	0.7267	0.9195	1.156	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.2648	0.5534	0.7176	0.9057	1.134	1.440	1.943	2.447	3.143	3.707	5.203	5.959
7	0.2632	0.5491	0.7111	0.8960	1.119	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.2619	0.5459	0.7064	0.8889	1.108	1.397	1.860	2.306	2.896	3.355	4.500	5.041
9	0.2610	0.5435	0.7027	0.8834	1.100	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.2602	0.5415	0.6998	0.8791	1.093	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.2596	0.5399	0.6974	0.8755	1.088	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.2590	0.5386	0.6955	0.8726	1.083	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.2586	0.5375	0.6938	0.8702	1.079	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.2582	0.5366	0.6924	0.8681	1.076	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.2579	0.5357	0.6912	0.8662	1.074	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.2576	0.5350	0.6901	0.8647	1.071	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.2573	0.5344	0.6892	0.8633	1.069	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.2571	0.5338	0.6884	0.8620	1.067	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.2569	0.5333	0.6876	0.8610	1.066	1.328	1.729	2.093	2.539	2.861	3.575	3.883
20	0.2567	0.5329	0.6870	0.8600	1.064	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.2566	0.5325	0.6864	0.8591	1.063	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.2564	0.5321	0.6858	0.8583	1.061	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.2563	0.5317	0.6853	0.8575	1.060	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.2562	0.5314	0.6848	0.8569	1.059	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.2561	0.5312	0.6844	0.8562	1.058	1.316	1.708	2.060	2.485	2.787	3.450	3.725
26	0.2560	0.5309	0.6840	0.8557	1.058	1.315	1.706	2.056	2.479	2.779	3.435	3.707
27	0.2559	0.5306	0.6837	0.8551	1.057	1.314	1.703	2.052	2.473	2.771	3.421	3.690
28	0.2558	0.5304	0.6834	0.8546	1.056	1.313	1.701	2.048	2.467	2.763	3.408	3.674
29	0.2557	0.5302	0.6830	0.8542	1.055	1.311	1.699	2.045	2.462	2.756	3.396	3.659
30	0.2556	0.5300	0.6828	0.8538	1.055	1.310	1.697	2.042	2.457	2.750	3.385	3.646
32	0.2555	0.5297	0.6822	0.8530	1.054	1.309	1.694	2.037	2.449	2.738	3.365	3.622
34	0.2553	0.5294	0.6818	0.8523	1.052	1.307	1.691	2.032	2.441	2.728	3.348	3.601
36	0.2552	0.5291	0.6814	0.8517	1.052	1.306	1.688	2.028	2.434	2.719	3.333	3.582
38	0.2551	0.5288	0.6810	0.8512	1.051	1.304	1.686	2.024	2.429	2.712	3.319	3.566
40	0.2550	0.5286	0.6807	0.8507	1.050	1.303	1.684	2.021	2.423	2.704	3.307	3.551
50	0.2547	0.5278	0.6794	0.8489	1.047	1.299	1.676	2.009	2.403	2.678	3.261	3.496
60	0.2545	0.5272	0.6786	0.8477	1.045	1.296	1.671	2.000	2.390	2.660	3.232	3.460
120	0.2539	0.5258	0.6765	0.8446	1.041	1.289	1.658	1.980	2.358	2.617	3.160	3.373
$\infty$	0.2533	0.5244	0.6745	0.8416	1.036	1.282	1.645	1.960	2.326	2.576	3.090	3.291

TABLE 12(b). 5 PER CENT POINTS OF THE *F*-DISTRIBUTION

If  $F = \frac{X_1}{\nu_1} / \frac{X_2}{\nu_2}$ , where  $X_1$  and  $X_2$  are independent random variables distributed as  $\chi^2$  with  $\nu_1$  and  $\nu_2$  degrees of freedom respectively, then the probabilities that  $F \geq F(P)$  and that  $F \leq F(P)$  are both equal to  $P/100$ . Linear interpolation in  $\nu_1$  and  $\nu_2$  will generally be sufficiently accurate except when either  $\nu_1 > 12$  or  $\nu_2 > 40$ , when harmonic interpolation should be used.



(This shape applies only when  $\nu_1 \geq 3$ . When  $\nu_1 < 3$  the mode is at the origin.)

$\nu_1 =$	1	2	3	4	5	6	7	8	10	12	24	$\infty$
$\nu_2 = 1$	161.4	199.5	215.7	224.6	230.2	234.0	236.8	238.9	241.9	243.9	249.1	254.3
2	18.51	19.00	19.16	19.25	19.30	19.33	19.35	19.37	19.40	19.41	19.45	19.50
3	10.13	9.552	9.277	9.117	9.013	8.941	8.887	8.845	8.786	8.745	8.639	8.526
4	7.709	6.944	6.591	6.388	6.256	6.163	6.094	6.041	5.964	5.912	5.774	5.628
5	6.608	5.786	5.409	5.192	5.050	4.950	4.876	4.818	4.735	4.678	4.527	4.365
6	5.987	5.143	4.757	4.534	4.387	4.284	4.207	4.147	4.060	4.000	3.841	3.669
7	5.591	4.737	4.347	4.120	3.972	3.866	3.787	3.726	3.637	3.575	3.410	3.230
8	5.318	4.459	4.066	3.838	3.687	3.581	3.500	3.438	3.347	3.284	3.115	2.928
9	5.117	4.256	3.863	3.633	3.482	3.374	3.293	3.230	3.137	3.073	2.900	2.707
10	4.965	4.103	3.708	3.478	3.326	3.217	3.135	3.072	2.978	2.913	2.737	2.538
11	4.844	3.982	3.587	3.357	3.204	3.095	3.012	2.948	2.854	2.788	2.609	2.404
12	4.747	3.885	3.490	3.259	3.106	2.996	2.913	2.849	2.753	2.687	2.505	2.296
13	4.667	3.806	3.411	3.179	3.025	2.915	2.832	2.767	2.671	2.604	2.420	2.206
14	4.600	3.739	3.344	3.112	2.958	2.848	2.764	2.699	2.602	2.534	2.349	2.131
15	4.543	3.682	3.287	3.056	2.901	2.790	2.707	2.641	2.544	2.475	2.288	2.066
16	4.494	3.634	3.239	3.007	2.852	2.741	2.657	2.591	2.494	2.425	2.235	2.010
17	4.451	3.592	3.197	2.965	2.810	2.699	2.614	2.548	2.450	2.381	2.190	1.960
18	4.414	3.555	3.160	2.928	2.773	2.661	2.577	2.510	2.412	2.342	2.150	1.917
19	4.381	3.522	3.127	2.895	2.740	2.628	2.544	2.477	2.378	2.308	2.114	1.878
20	4.351	3.493	3.098	2.866	2.711	2.599	2.514	2.447	2.348	2.278	2.082	1.843
21	4.325	3.467	3.072	2.840	2.685	2.573	2.488	2.420	2.321	2.250	2.054	1.812
22	4.301	3.443	3.049	2.817	2.661	2.549	2.464	2.397	2.297	2.226	2.028	1.783
23	4.279	3.422	3.028	2.796	2.640	2.528	2.442	2.375	2.275	2.204	2.005	1.757
24	4.260	3.403	3.009	2.776	2.621	2.508	2.423	2.355	2.255	2.183	1.984	1.733
25	4.242	3.385	2.991	2.759	2.603	2.490	2.405	2.337	2.236	2.165	1.964	1.711
26	4.225	3.369	2.975	2.743	2.587	2.474	2.388	2.321	2.220	2.148	1.946	1.691
27	4.210	3.354	2.960	2.728	2.572	2.459	2.373	2.305	2.204	2.132	1.930	1.672
28	4.196	3.340	2.947	2.714	2.558	2.445	2.359	2.291	2.190	2.118	1.915	1.654
29	4.183	3.328	2.934	2.701	2.545	2.432	2.346	2.278	2.177	2.104	1.901	1.638
30	4.171	3.316	2.922	2.690	2.534	2.421	2.334	2.266	2.165	2.092	1.887	1.622
32	4.149	3.295	2.901	2.668	2.512	2.399	2.313	2.244	2.142	2.070	1.864	1.594
34	4.130	3.276	2.883	2.650	2.494	2.380	2.294	2.225	2.123	2.050	1.843	1.569
36	4.113	3.259	2.866	2.634	2.477	2.364	2.277	2.209	2.106	2.033	1.824	1.547
38	4.098	3.245	2.852	2.619	2.463	2.349	2.262	2.194	2.091	2.017	1.808	1.527
40	4.085	3.232	2.839	2.606	2.449	2.336	2.249	2.180	2.077	2.003	1.793	1.509
60	4.001	3.150	2.758	2.525	2.368	2.254	2.167	2.097	1.993	1.917	1.700	1.389
120	3.920	3.072	2.680	2.447	2.290	2.175	2.087	2.016	1.910	1.834	1.608	1.254
$\infty$	3.841	2.996	2.605	2.372	2.214	2.099	2.010	1.938	1.831	1.752	1.517	1.000